Berkovich spaces over $\mathbb{Z}$ and Schottky spaces

Abstract: Berkovich spaces over $\mathbb{Z}$ look like fibrations that contain complex analytic spaces as well as $p$-adic analytic spaces for every prime number $p$. We will give a short introduction to those spaces and explain that they provide a convenient setting to parametrize certain natural families such as Schottky groups and Mumford curves over arbitrary local fields. More precisely, for each $g$ greater than 1, there exists an open subset $S_g$ of the affine analytic space over $\mathbb{Z}$ of dimension $3g - 3$ that parametrizes Schottky groups with $g$ generators. Using Schottky uniformization (due to Mumford in the non-archimedean case), one can associate with each point of $S_g$ a projective curve, obtained as a quotient of an open subset of the projective line by the action of the corresponding Schottky group. We will explain that it is possible to carry out this construction globally: there is a universal Mumford curve that is projective over $S_g$ as well as a universal uniformization over $S_g$ with a globally defined analytic map to the universal curve. (This is joint work with Daniele Turchetti.)